## Expected values of Discrete Random variables

Expected values of random variables is denoted by $E(x)$

$E(x) = \sum x_i \times P(x_i)$ where $x_i$ = discontinuous set of values

$P(x_i)$ = probability of discontinuous set of values

$= x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + \dots x_n P(x_n)$

variance of discrete random variable

$$V[x] = \sum E(x_i^2) - [\sum E(x_i)]^2$$

$$\sum E(x_i^2) = P(x_1) \times x_1^2 + P(x_2) \times x_2^2 + \dots + P(x_n) \times x_n^2$$

## Properties of Expected values and variances

There are following properties of expected value and variances.

(a) The expected values of a constant c is constant of mean, $E(c) = c$ for every constant.

(b) The expected values of the product of a constant c and a random variable x is equal to constant c time the expected value of the random variable
$$E[cx] = c[E(x)]$$

(c) The expected value of linear function of a random variable is same as the linear function of its expectation. $\Rightarrow E(a + bx) = a + b E(x)$

(d) The expected value of the product of two independent variables is equal to the product of their individual expected values.
$$E(xy) = E(x) \times E(y)$$
(f) $var[cx] = c^2 var(x)$

(e) $E[x+y] = E[x] + E[y]$

(g) variance $[x \pm y] = var(x) \pm var(y)$

Q → Under an employment promotion programme it is proposed to allow sale of newspapers inside buses during off peak hours. The vendors can purchase newspapers at a special concessional rate of Rs 1.25 per copy against the selling price of Rs 1.50. Any unsold copies are however, a dead loss.

A vendor has estimated the following probability distribution for the number of copies demanded

| Number of copies | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|
| Probability | 0.04 | 0.19 | 0.33 | 0.26 | 0.11 | 0.07 |

How many copies should be ordered so that his expected profit will be maximum.

Solution :→ step I - at first we have to calculate profit per copy.

Profit per copy = selling price − purchasing price
= 1.50 − 1.25
= 0.25

step 2. we have to calculate the Expected profit for each demand of the copies.

Expected profit = Number of copies × probability × profit per copy

| Number of copies demanded | probability | profit/copy | Expected profit |
|---|---|---|---|
| 15 | 0.04 | 0.25 | $E(P) = 15 \times 0.04 \times 0.25 = 0.15$ |
| 16 | 0.19 | 0.25 | $16 \times 0.19 \times 0.25 = 0.76$ |
| 17 | 0.33 | 0.25 | $17 \times 0.33 \times 0.25 = 1.40$ |
| 18 | 0.26 | 0.25 | $18 \times 0.26 \times 0.25 = 1.17$ |
| 19 | 0.11 | 0.25 | $19 \times 0.11 \times 0.25 = 0.52$ |
| 20 | 0.07 | 0.25 | $20 \times 0.07 \times 0.25 = .35$ |

Q- A company introduces a new product in the market and expect to make a profit of Rs 2.5 lakh during the first year if the demand is good: Rs 1.5 lakh, if the demand is moderate and a loss of 1 lakh if the demand is 'poor'. Market research studies indicate that the probabilities for the demand to be good and moderate are 0.2 and 0.5 respectively. Find the company expected profit and the standard deviation.

Solution; → Step 1. Suppose that $x_1, x_2$ and $x_3$ are the profit in three different condition like good demand, moderate demand and poor demand.

From question, we know that that the probabilities of good demand and moderate

$P[x_1] = 0.2$

$P[x_2] = 0.5$

Step 2 we have to find the value of $P[x_3]$

Since these events are mutually exclusive and exhaustive,

therefore, $P[x_1] + P[x_2] + P[x_3] = 1$

$0.2 + 0.5 + P[x_3] = 1$

$P[x_3] = 0.3$

Step 3 Expected profit $= x_1 \times P[x_1] + x_2 \times P[x_2] + x_3 \times P[x_3]$

where $x_1, x_2, x_3$ are the profit in good, moderate and poor demand

$P[x_1], P[x_2], P[x_3]$ = probabilities of $x_1, x_2$ and $x_3$ in three different condition

$E[X] = 2.5 \times 0.2 + 1.5 \times 0.5 - 1 \times 0.3$

$= 0.5 + 0.75 - 0.3$

$= 0.95$

$SD(x) = \sqrt{1.7725}$

$= 1.331$ lakh

$V(X) = E[X^2] - (E[X])^2$

$= (2.5)^2 \times 0.2 + (1.5)^2 \times 0.5 + (-1)^2 \times 0.3 - (0.95)^2$

$= 1.25 + 1.125 + 0.3 - .9025 = 1.7725$

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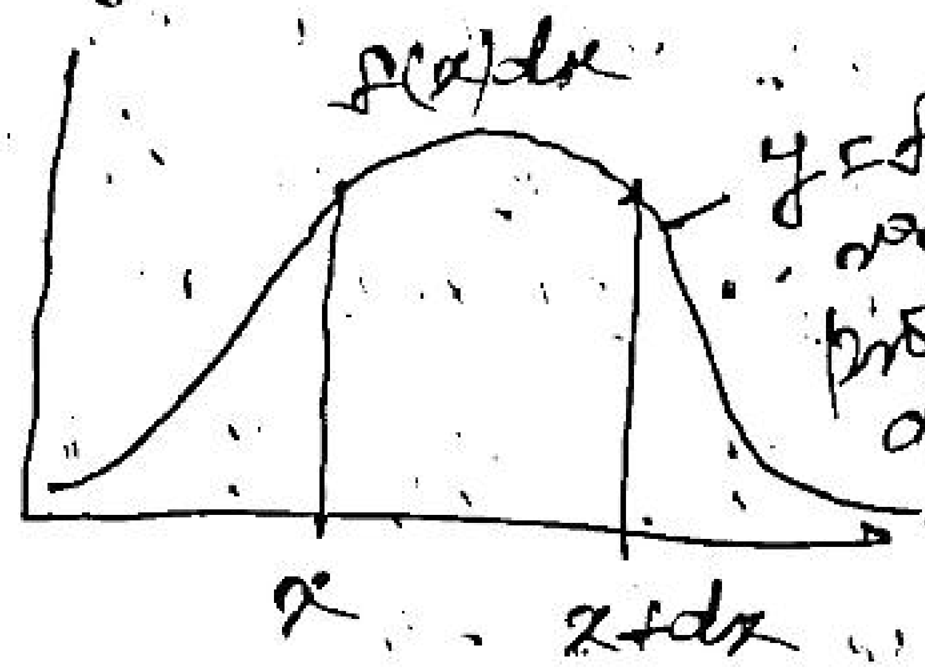
# Probability density function of continuous random variable functions

def: we consider a small interval $(x, x+dx)$ of length $dx$

$f(x)$ is the continuous function of $x$

$f(x)dx$ represents probability X lies between in infinite small interval $(x, x+dx)$




$P(x \le X \le x+dx) = f(x)\,dx$

$P(a \le x \le b) = \int_a^b f(x)\,dx$ in the condition finite numbers

$P(-\infty \le x \le \infty) = \int_{-\infty}^{\infty} f(x)\,dx$

Properties of Probability density function

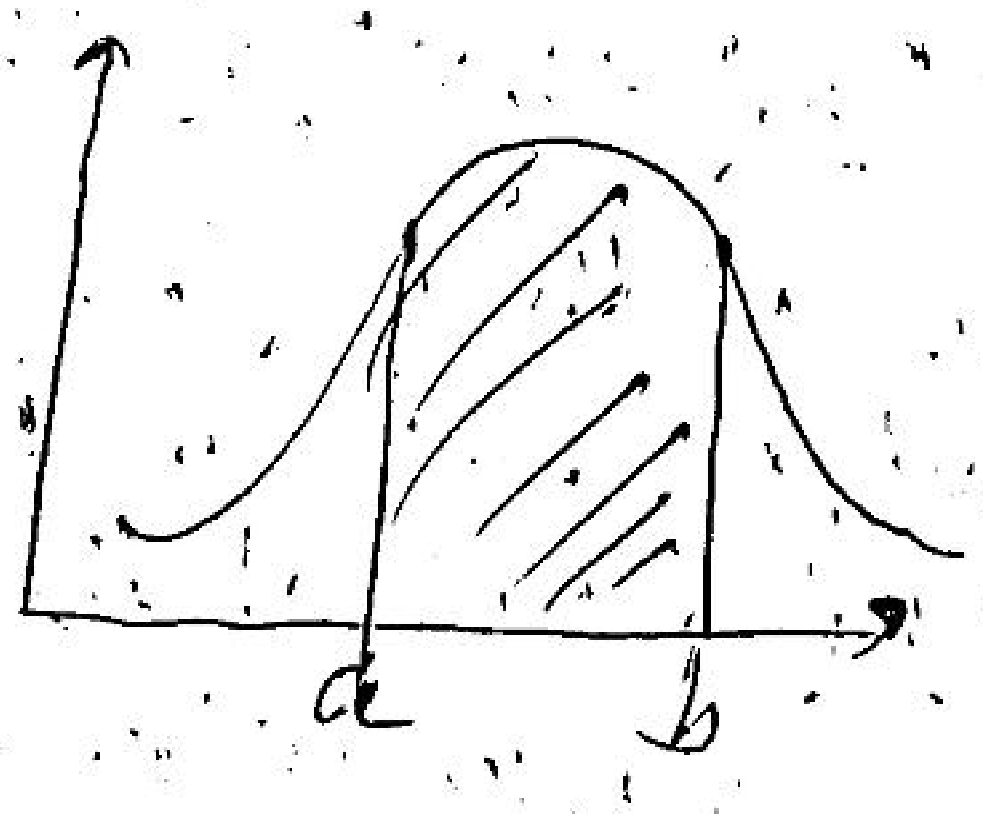
1. $f(x) \ge 0$
2. $\int_{-\infty}^{\infty} f(x)\,dx = 1$
3. $P(E) \quad \int_E f(x)\,dx$

For the probability density function we can not calculate the pdf of single point

$a = b$

$\int_a^a f(x)\,dx = 0$

$P(a \le x \le a) = \int_a^a f(x)\,dx = 0$




$f(x) = \phi(x)$

$f(x) = 0$ when $x < a$

$f(x) = \phi(x) \quad a \le x \le b$

$f(x) = 0 \quad x > b$

Q. If $f(x)$ is a continuous random variable with the following pdf

$$f(x) = \alpha(2x - x^2) \quad 0 < x < 2$$
$$\quad\quad 0 \quad \text{otherwise}$$

find (i) $\alpha$
(ii) $P(x > 1)$

Solution: From the definition of pdf

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx + \int_{2}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} f(x)\,dx = 0 \qquad \int_{2}^{\infty} f(x)\,dx = 0$$

$$\int_{0}^{2} f(x)\,dx = 1$$

$$\Rightarrow \int_{0}^{2} \alpha(2x - x^2)\,dx = 1$$

$$\Rightarrow \alpha\left[x^2 - \frac{x^3}{3}\right]_0^2 = 1$$

$$\Rightarrow \cancel{\alpha\left[4 - \frac{4}{3}\right] = 1}$$

$$\Rightarrow \cancel{\alpha = \frac{1}{2}}$$

$$\Rightarrow \alpha\left[x^2 - \frac{x^3}{3}\right]_0^2 = 1$$

$$\Rightarrow \alpha\left[4 - \frac{8}{3}\right] = 1$$

$$\Rightarrow \alpha \times \frac{4}{3} = 1$$

$$\alpha = \frac{3}{4}$$

$$P(x > 1) = \int_{1}^{\infty} f(x)\,dx$$

$$= \alpha \int_{1}^{2} (2x - x^2)\,dx$$

$$= \cancel{\alpha\left[x^2 - \frac{x^3}{3}\right]}$$

$$= \alpha\left[x^2 - \frac{x^3}{3}\right]_1^2$$

$$= \frac{3}{4}\left[\left(2^2 - \frac{2^3}{3}\right) - \left(1 - \frac{1}{3}\right)\right]$$

$$= \frac{3}{4}\left[\left(4 - \frac{8}{3}\right) - \left(1 - \frac{1}{3}\right)\right]$$

$$= \frac{3}{4}\left[\frac{4}{3} - \frac{2}{3}\right]$$

$$= \frac{3}{4} \times \frac{2}{3}$$

$$= \frac{1}{2}$$

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Q. A random variable $x$ has density function

$$f(x) = \begin{cases} kx^2 & -3 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

find $P(1 \le x \le 2)$, $P(x > 1)$, $P(x \le 2)$

From the definition, we know that

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_{-3}^{3} kx^2\,dx = 1$$

$$\Rightarrow k\int_{-3}^{3} x^2\,dx = 1$$

$$\Rightarrow k\left[\frac{x^3}{3}\right]_{-3}^{3} = 1$$

$$\Rightarrow k\left[\left(\frac{27}{3}\right) - \left(-\frac{27}{3}\right)\right] = 1$$

$$\Rightarrow k\left(\frac{27}{3} + \frac{27}{3}\right) = 1$$

$$k \times \frac{54}{3} = 1$$

$$\Rightarrow k = \frac{1}{18}$$

$$P(1 \le x \le 2) = \int_1^2 f(x)\,dx$$

$$= k\int_1^2 x^2\,dx$$

$$= \frac{1}{18}\left[\frac{x^3}{3}\right]_1^2$$

$$= \frac{1}{18}\left(\frac{8}{3} - \frac{1}{3}\right)$$

$$= \frac{1}{18} \times \frac{7}{3}$$

$$= \frac{7}{54}$$

$$P(x \le 2) = \int_{-\infty}^{2} f(x)\,dx = \int_{-3}^{2} kx^2\,dx$$

$$= k\left[\frac{x^3}{3}\right]_{-3}^{2}$$

$$= \frac{1}{18}\left(\frac{8}{3} + \frac{27}{3}\right)$$

$$= \frac{1}{18} \times \frac{35}{3}$$

$$= \frac{35}{54}$$

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$$P(X \geq 1) = \int_1^3 f(x)\,dx.$$

$$= \int_1^3 x^2\,dx$$

$$= \frac{1}{18}\int_1^3 x^2\,dx$$

$$= \frac{1}{18}\left[\frac{x^3}{3}\right]_1^3$$

$$= \frac{1}{18}\left[\frac{27}{3} - \frac{1}{3}\right]$$

$$= \frac{26}{54}.$$

Commulative density function (cdf)

Let $x$ be a continuous random variable having pdf $f(x)$ then $F_x(x)$ will be continuous distribution function of $x$ if

$$\boxed{F_x(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx}$$

This is also known as commulative distribution function.

Relation between Distribution function and density function.

$$\frac{dF(x)}{dx} = f(x)$$

properties of ~~commu~~ Commulative distribution function

① $0 \leq F(x) \leq 1$

② $\frac{d}{dx}F(x) = F'(x) = f(x) \geq 0$

③ $F(-\infty) = \lim_{x \to -\infty} \int_{-\infty}^{x} f(x)\,dx = 0$

④ $F(\infty) = \lim_{x \to \infty} \int_{-\infty}^{x} f(x)\,dx$
$\int_{-\infty}^{\infty} f(x)\,dx = 1$

Q -> $c = ?$ for pdf

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$$f(x) = cx^2 \quad 0 \le x \le 3$$
$$= 0 \quad \text{otherwise}$$

and compute $P(1 \le x \le 2)$

also find cdf.

From the definition of pdf

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_{0}^{3} f(x)\,dx = 1$$

$$\Rightarrow \int_{0}^{3} cx^2\,dx = 1$$

$$\Rightarrow c\int_{0}^{3} x^2\,dx = 1$$

$$\Rightarrow c\left[\frac{x^3}{3}\right]_0^3 = 1$$

$$\Rightarrow 9c = 1$$

$$\Rightarrow c = \frac{1}{9}$$

$$f(x) = \frac{x^2}{9} \quad 0 \le x \le 3$$
$$= 0 \quad \text{otherwise}$$

In the second question, we have to find out the cdf.

then we know that cdf

$$F(x) = P[X \le x] = \int_{-\infty}^{x} \frac{x^2}{9}$$

$$= \left[\frac{x^3}{27}\right]_0^x$$

$$= \frac{x^3}{27}$$

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$F(x) = 0 \quad x < 0$

$F(x) = \frac{x^3}{27} \quad 0 \le x \le 3$

$F(x) = 1, \quad x \geq 3$

$P(1 \le x \le 2) = \int_{-\infty}^{\infty} f(x)\,dx$

$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx$

$= \int_{1}^{2} f(x)\,dx$

$= \int_{1}^{2} \frac{x^2}{9}\,dx$

$= \left[\frac{x^3}{27}\right]_1^2$

$= \frac{8}{27} - \frac{1}{27}$

$= \frac{7}{27}$

Expected value and variance of continuous random variable

$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

$var(x) = E(x^2) - [E(x)]^2$

$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

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Q. = $f(x) = \frac{3}{10}(3x - x^2)$ $0 \le x \le 2$ (10)

$= 0$ otherwise

then find the mean and variance.

Solution → for finding the value of variance we have to calculate the values of mean [E(X)] and $E(X^2)$

From the definition we know that

$$E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{-\infty}^{0} x f(x)\, dx + \int_{0}^{2} x f(x)\, dx$$

$$= \int_{0}^{2} x f(x)\, dx$$

$$= \int_{0}^{2} x \frac{3}{10}(3x - x^2)\, dx$$

$$= \frac{3}{10}\left[\int_{0}^{2} 3x^2\, dx - \int_{0}^{2} x^3\, dx\right]$$

$$= \frac{3}{10}\left[\left(3 \times \frac{x^3}{3}\right)_0^2 - \left(\frac{x^4}{4}\right)_0^2\right]$$

$$= \frac{3}{10}\left[(x^3)_0^2 - \left(\frac{x^4}{4}\right)_0^2\right]$$

$$= \frac{3}{10}[8 - 4]$$

$$= \frac{3}{10} \times 4$$

$$= \frac{6}{5}$$

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$$E(x^2) = \int_0^2 x^2 \times \frac{3}{10}(3x - x^2)\,dx \quad (1)$$

$$= \frac{3}{10}\left[\int_0^2 3x^3\,dx - \int_0^2 x^4\,dx\right]$$

$$= \frac{3}{10}\left[\frac{3}{4}\left[x^4\right]_0^2 - \frac{1}{5}\left(x^5\right)_0^2\right]$$

$$= \frac{3}{10}\left[\frac{3}{\cancel{4}} \times \cancel{16}^{4} - \frac{1}{5} \times 32\right]$$

$$= \frac{3}{10}\left(12 - \frac{32}{5}\right)$$

$$= \frac{3}{\cancel{10}_{5}} \times \frac{\cancel{28}^{14}}{5}$$

$$= \frac{42}{25}$$

$$\text{variance} = E(x^2) - [E(x)]^2$$

$$= \frac{42}{25} - \left(\frac{6}{5}\right)^2$$

$$= \frac{42}{25} - \frac{36}{25}$$

$$= \frac{6}{25}$$

Q → The probability density function for random variable $x$ is given by

$$f(x) = \begin{cases} \frac{x}{4} & 0 \le x \le 2 \\ \frac{1}{4}(4-x) & 2 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

Find the mean and standard deviation of $x$

From the definition,
we know that

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{-\infty}^{0} x f(x)\, dx + \int_{0}^{2} x f(x)\, dx + \int_{2}^{4} x f(x)\, dx$$

$$= \int_{0}^{2} x f(x)\, dx + \int_{2}^{4} x f(x)\, dx$$

$$= \int_{0}^{2} x \times \frac{x}{4}\, dx + \int_{2}^{4} x \times \frac{(4-x)}{4}\, dx$$

$$= \frac{1}{4}\int_{0}^{2} x^2\, dx + \frac{1}{4}\int_{2}^{4} (4x - x^2)\, dx$$

$$= \frac{1}{4}\left[\frac{x^3}{3}\right]_0^2 + \frac{1}{4}\left[4 \times \frac{x^2}{2} - \frac{x^3}{3}\right]_2^4$$

$$= \frac{1}{\cancel{4}} \times \frac{\cancel{8}^{2}}{3} + \frac{1}{4}\left[\left(2 \times 16 - \frac{64}{3}\right) - \left(8 - \frac{8}{3}\right)\right]$$

$$= \frac{2}{3} + \frac{1}{4}\left[\left(\frac{96-64}{3}\right) - \frac{16}{3}\right]$$

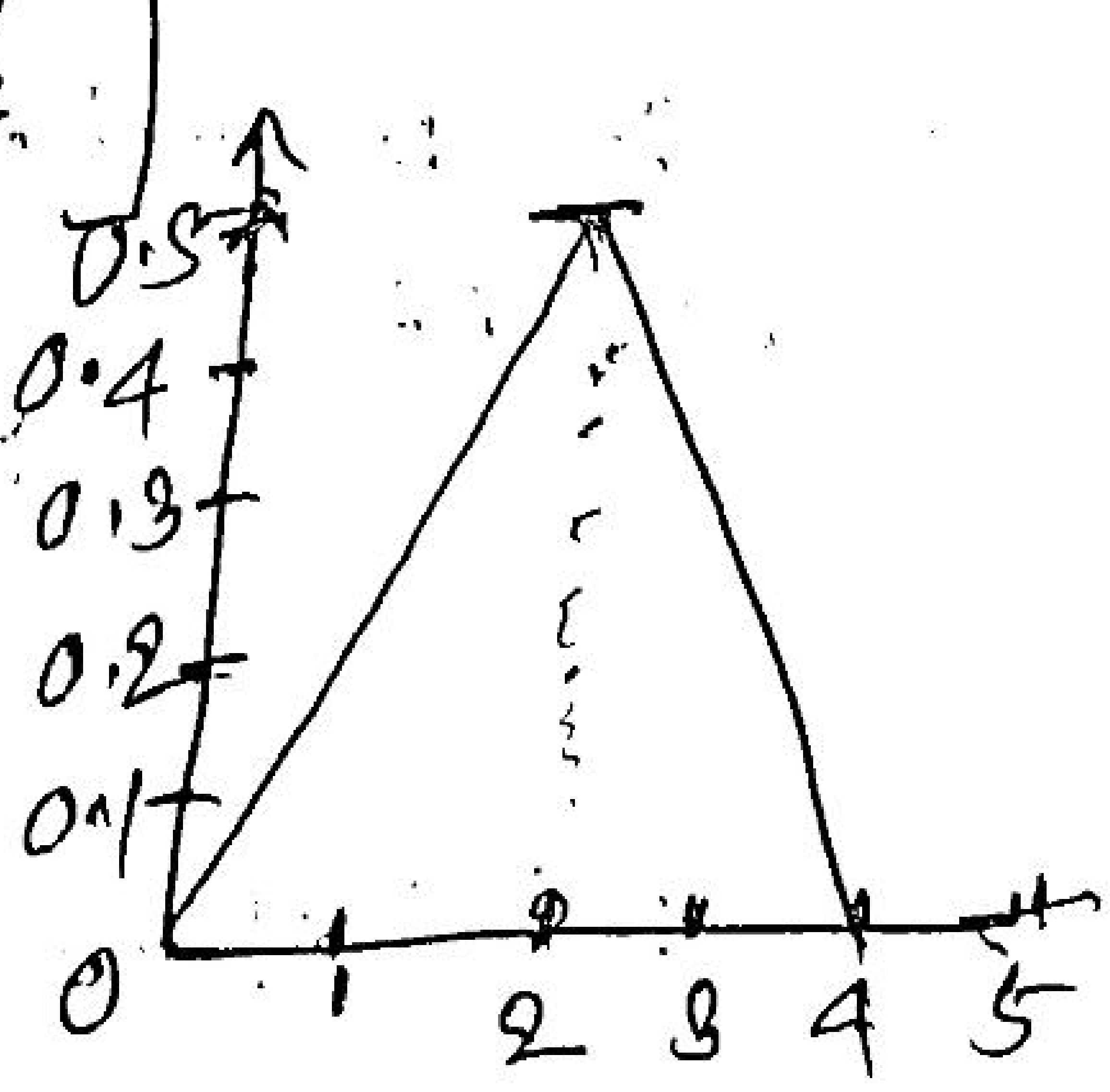
$$= \frac{2}{3} + \frac{1}{4}\left[\frac{32}{3} - \frac{16}{3}\right]$$

$$= \frac{2}{3} + \frac{1}{\cancel{4}} \times \frac{\cancel{16}^{4}}{3}$$

$$= \frac{2}{3} + \frac{4}{3}$$

$$= \frac{6}{3}$$

$$= 2$$

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$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$= \int_{-\infty}^{0} x^2 f(x)\,dx + \int_{0}^{2} x^2 f(x)\,dx + \int_{2}^{4} x^2 f(x)\,dx$$

$$= \int_{0}^{2} x^2 \times \frac{x}{4}\,dx + \int_{2}^{4} \frac{x^2(4-x)}{4}\,dx$$

$$= \frac{1}{4}\int_{0}^{2} x^3\,dx + \frac{1}{4}\left[\int_{2}^{4} (4x^2 - x^3)\,dx\right]$$

$$= \frac{1}{4}\left[\frac{x^4}{4}\right]_0^2 + \frac{1}{4}\left[\frac{4x^3}{3} - \frac{x^4}{4}\right]_2^4$$

$$= \frac{1}{\cancel{4}} \times \frac{\cancel{16}^{4}}{\cancel{4}} + \frac{1}{4}\left[\left(\frac{256}{3} - \frac{256}{4}\right) - \left(\frac{32}{3} - \frac{\cancel{16}^{4}}{\cancel{4}}\right)\right]$$

$$= 1 + \frac{1}{4}\left(\frac{256}{12} - \left(\frac{128 - 48}{12}\right)\right)$$

$$\cancel{= 1 + \frac{1}{4}\left(\frac{256}{12} - \frac{80}{12}\right)}$$

$$\cancel{= 1 + \frac{1}{4}\left(\frac{256 - 240}{12}\right) = 1 + \frac{1}{4} \times \frac{256}{12}}$$

$$\cancel{= 1 + \frac{1}{4} \times \frac{64}{3}}$$

$$\cancel{= \frac{7}{3}}$$

$$= 1 + \frac{1}{4}\left(\frac{256}{12} - \left(\frac{32 - 12}{3}\right)\right)$$

$$= 1 + \frac{1}{4}\left(\frac{64}{3} - \frac{20}{3}\right)$$

$$= 1 + \frac{1}{4} \times \frac{44}{3} = 1 + \frac{11}{3} = \frac{14}{3}$$

variance $= E(x^2) - [E(x)]^2$

$$= \frac{14}{3} - 4$$

$$\sigma^2 = \frac{2}{3}, \quad \sigma = \sqrt{\frac{2}{3}}$$